CHAPTER 12

PROBABILITY

Background of Chapter

Conceptual Background

- Basic Definition
- Random Experiment
- Events
  - Sure event & Impossible Event
  - Mutually Exclusive Events
  - Exhaustive Events
  - Independent Event
- Classical Definition Of Probability
- Sample Space
- Calculation of Outcomes

Question Background

- Phase-1
  - Single Event
  - More than One Event
  - Total Probability
- Phase-2
  - Conditional Probability
  - Expected Value
  - Common Algebraic Problems
  - Odds In Favor Odds Against

Basic Definition

Probability means **Chance** of "Occurrence" of any Particular Event

Experiment

Deterministic Experiment

Experiment Whose Outcome is Known is Known as **Deterministic Experiment**

E.g.: Solution Prepared in LAB

Random Experiment

Experiment Whose Outcome is Known is not Known as **Random Experiment**

E.g.: Tossing a Coin, Rolling a Dice

Events

It is always the Subset of Experiment

Sure Event

$P(A) = 1$

Impossible Event

$P(A) = 0$

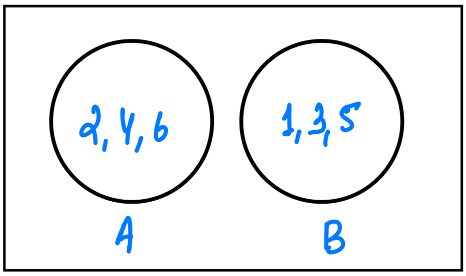
Equiprobable Event / Equally Likely Event

Two events are said to be equiprobable, if they have ***same chance*** *of* ***Occurrence.***

if $\boxed{P(A) = P(B)}$ $\Rightarrow$ then A & B are equiprobable Events.

Mutually Exclusive Events

Two events are said to ***Mutually Exclusive,*** *if they have* ***Nothing Common***



E.g. Throwing a Dice: {1,2,3,4,5,6}.

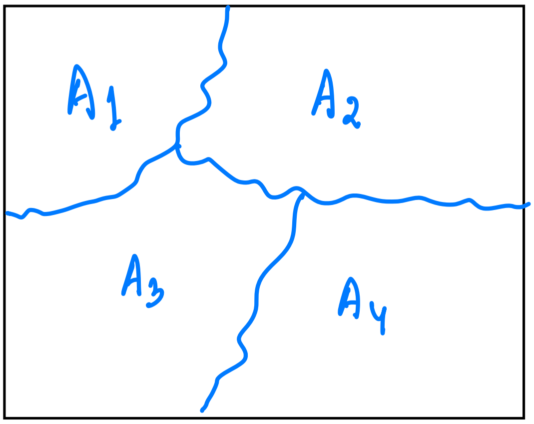
Event (A): Getting **Even** No - $\{2, 4, 6\}$

Event (B): Getting **Odd** No - $\{1, 3, 5\}$

A & B are M.E.

Exhaustive Events

*Set of Events, **Union [+]** of which will give us **entire Sample Space** is Known as **Exhaustive Events.***



$A_1 + A_2 + A_3 + A_4$ = Sample Space.

E.g. Throwing a Dice: {1,2,3,4,5,6}.

Event (A): Getting **Even** No - $\{2, 4, 6\}$
Event (B): Getting **Odd** No - $\{1, 3, 5\}$

$\left.\right\}$ Exhaustive events.

$A + B = \{1,2,3,4,5,6\} \rightarrow$ S.S.

Independent Events

We will discuss this in G.M.T.

Classical Definition of Probability

If there are **"n"** Possible outcomes and out of which **"m"** possible cases favors happening of event A, then according to Classical Definition of probability

$$P[A] = \frac{m}{n} = \frac{\textit{no of elements in favour of A}}{\textit{Total no of Outcomes}}$$

NOTE: Favorable cases**: m,** Total no of Cases **= n**

Let's Draw Some Conclusion's

1. Non-Favorable Cases **= n-m**
2. $0 \leq m \leq n$

   Divide by "n"

   $\frac{0}{n} \leq \frac{m}{n} \leq \frac{n}{n}$
3. $\boxed{0 \leq P[A] \leq 1}$ (Impossible event → 0, Sure Event → 1)
4. $\boxed{P(A) + P(\bar{A}) = 1}$

Conclusion Note

Student: Sir can we conclude from above that probability calculation in based on **NO OF OUTCOMES**

AK Sir: Yes Of Course we need to two things **No of favorable Outcomes** as well as **Total no of Outcomes**

- **Sample Space**
- **Use of Combination**

Sample Space

All the Possible **Outcomes** of an experiment is known as Sample Space

E.g., Tossing of 2 Coins
Sample Space: {HH, HT, TH, TT}

E.g., Rolling a dice
Sample Space: {1,2,3,4,5,6}

NOTE

We will Create Sample Space in question if it small and possible …So let's **Learn** Some

Basic Sample Space

- Coins
- Dice
- Cards
- ∞

Coins Sample Space: **No of Outcomes =** $(\text{Outcomes of Single Coin})^{\text{No of Coins}}$

1 Coin = 2

{H, T}

2 Coin

$(2)^2 = 4$

{HH}
{TH}
{HT}
{TT}

3 Coin

$(2)^3 = 8$

{HHT} {HHH}
{HTT} {HTH}
{TTT} {THT}
{TTH}
{THH}

Dice Sample Space: **No of Outcomes =** $(Outcomes\ of\ Single\ Dice)^{No\ of\ Dices}$

1 Dice $= 6$

$\{1, 2, 3, 4, 5, 6\}$

2 Dice $\Rightarrow (6)^2 = 36$

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Example

If two dice is rolled find the probability of getting **(a)** Sum as 2 **(b)** Sum as 5

(a) $P(2) = \frac{fav}{Total} = \frac{1}{36}$ (b) $P(5) = \frac{4}{36}$

Shortcut

To get the favorable No of Cases or Probability in case of 2 Dice and Sum is Demanded

$P(sum)$:

- $\frac{(x-1)}{36}$ $\rightarrow fav$, $2 \leq x \leq 7$
- $\frac{13-x}{36}$ $\rightarrow fav$, $8 \leq x \leq 12$

Example: Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4?

Cards Sample Space

52 Cards (Joker is not a Card)

BLACK CARDS [26 Cards]	
SPADE [13]	CLUB [13]
A	A
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
J	J
Q	Q
K	K

Denomination Cards.

Face Cards.

RED CARDS [26 Cards]	
HEART [13]	DIAMOND [13]
A	A
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
J	J
Q	Q
K	K

NOTE

Face Cards = $3 \times 4 = 12$

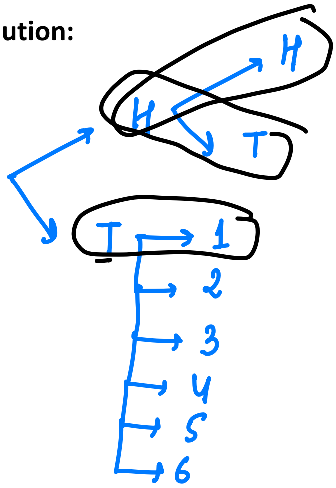
Honor Cards = Ace + Face Cards $= 4 \times 4 = 16$

Whenever there is a Mixture of Events and Question Demands Sample Space → **Tree Diagram Approach**

Example

A fair coin is tossed, if it shows Head, then again, a coin is flipped, and if it shows Tail, then a dice is rolled. Find the sample space and let me know Total No of Outcomes?

Solution:



Sample Space.

(HH) (HT) (T,1) (T,2) (T,3) (T,4) (T,5) (T,6)

= 8 Outcomes.

Students usually get Confused how to Compute
No of Cases

Sample Space

When the Data is Small *and it is Practically* ***Possible*** *to* ***Count*** *No of Outcomes* Manually *in that case prefer* ***Sample Space.***

E.g. One card is drawn from pack of 52 cards, find the probability of getting king.

$P(A) = \frac{F}{T} = \frac{4}{52}$

Use of Permutation & Combination

When the Data is Large *and it is* ***not*** *Practically* ***Possible*** *to* ***draw*** *Sample Space in that case prefer* ***Combination.***

E.g. Two cards are drawn from pack of 52 cards, find the probability of getting Both kings.

$P(E) = \frac{F}{T} = \frac{{}^4C_2}{{}^{52}C_2} = \frac{6}{1326}$

Example

If 2 Alphabets are selected at Random from a word "**MOBILE**", What is the probability that alphabets are Vowels?

Sample Space Technique

(MO, MB, MI, ML, ME, OB, OI
OL, OE, BI, BL, BE, IL, IE, LE)

$P(A) = \frac{F}{T} =$

Combination Technique

$P(A) = \frac{F}{T} =$

Answer the following which Technique is best in following Situations

S.no	Situation	Technique	No of Outcomes
1.	If 1 Alphabets are selected at Random from a word "**MOBILE**", What is the probability that alphabets is Vowels?		
2.	If 2 Alphabets are selected at Random from a word "**MOBILE**", What is the probability that alphabets are Vowels?		
3.	One card is drawn from a pack of 52 cards. What is the probability that it is red Card?		
4.	Two cards are drawn from a pack of 52 cards. What is the probability that both are red?		
5.	A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?		
6.	A bag contains 12 balls which are numbered from 1 to 12. If 3 ball are selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?		
7.	A bag contains 8 red and 5 white balls. A ball is drawn at random. The probability ball drawn is a red ball.		
8.	A bag contains 8 red and 5 white balls. Two balls are drawn at random. The probability ball drawn are red ball.		
9.	A bag consists of 5 red and 4 white balls, 4 balls are drawn at random what is the probability that 2 are red and 2 are White?		
10.	Probability that Hameed passes in mathematics is $\frac{2}{3}$ and the probability that he passes in English is $\frac{4}{9}$. If the probability of passing both courses is $\frac{1}{4}$ what is the probability that Hameed will pass in at least one of these subjects?		

Phase-1
Clarification Note

Till now we have solved all questions which were based on **SINGLE EVENT** and now have to move towards questions consisting of **MORE THAN ONE EVENT**

But after sometimes students get confused when to think for Theorems and When NOT, Lets Understand it with help of **CHART** below

A Single Event

We have **2 Alternatives** to Solve Question

$C.D.P = \frac{Fav}{Total}$

When it is Possible to "Count" outcomes then we Prefer → C.D.P.

Sample space

Combinatorics

General Addition Theorem **(GAT)**

It is not Possible to Count "Outcomes as Question has Given Probabilities Directly.

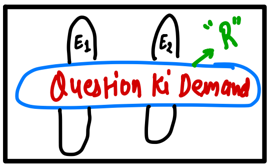
B More than ONE Event

General Multiplication Theorem **(GMT)**

How to Identify Single or Multiple Event

- In Succession.
- Two Consecutive draws.
- One by One.
- And another draws.
- with Replacement | without Replacement

C Total Probability



$P(R) = P[B_1 \cap R] + P[B_2 \cap R]$

Whenever there are Many events which fulfills the demands of the Question

we will calculate the Probability of each Event Separately and Add them to get Total Probability.

A bag consists of 5 red and 4 white balls, **4 balls are drawn at random** what is the probability that 2 are red and 2 are White?

$\boxed{\begin{matrix}5R \\ 4W\end{matrix}}$

A bag consists of 5 red and 4 white balls **2 Successive drawn of 2 balls** are made what is the probability that first 2 are red and another 2 are White?

$\boxed{\begin{matrix}5R \\ 4W\end{matrix}}$

A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. **One bag is selected at random.** from the selected bag, **one ball is drawn.** Find the probability that the ball drawn is **red**.

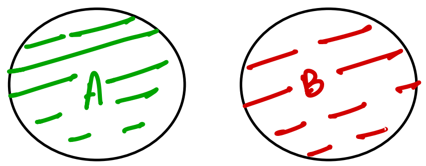
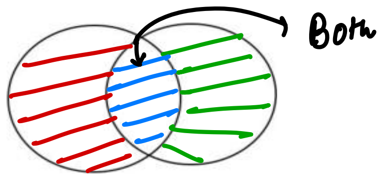
$\boxed{\begin{matrix}4R \\ 3B\end{matrix}}$ $\boxed{\begin{matrix}2R \\ 4B\end{matrix}}$

Theorems on Probability

General Addition Theorem (GAT)

When we want ONE or MORE Than ONE Task to occur {atleast one task to occur} **OR** → "∪"

$P(A \cup B) = P(A\ Or\ B) = P(A + B) = P(atleast\ one\ event\ to\ occure) = P(either\ A\ or\ B)$

When Events are Mutually exclusive	When Events are NOT Mutually exclusive
	
$P(A \cup B) = P(A) + P(B)$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Category of Manipulation in GAT

$$P(A \cup B) = P(A) + P(B) - \boxed{P(A \cap B)}$$

If events are Independent → Apply "GMT" → $P(A \cap B) = P(A) \times P(B)$

1 We Can **Count** No of Favourable Cases of Intersection of A & B

Example: A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?

Solu:

CDP: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

$P(A) = \frac{F}{T} =$

GAT

$P(5 \cup 6) = P(5) + P(6) - P(5 \cap 6)$

$\frac{2}{12} + \frac{2}{12} - 0$

$= \frac{4}{12}$

2 **Direct Probability** of Intersection of A & B is Given

Example: Probability that Hameed passes in mathematics is $\frac{2}{3}$ and the probability that he passes in English is $\frac{4}{9}$. If the probability of passing both courses is $\frac{1}{4}$ what is the probability that Hameed will pass in at least one of these subjects?

Solu:

$P(M) = \frac{2}{3}$, $P(E) = 4/9$, $P(M \cap E) = 1/4$

GAT

$P(M \cup E) = P(M) + P(E) - P(M \cap E)$

$\frac{2}{3} + \frac{4}{9} - 1/4$

$= \frac{24+16-9}{36} = \frac{31}{36}$

3 **Neither** We Can **Count nor Direct Probability is Given**

Example: The Probability that a football team winning a match at Kolkata is $\frac{3}{5}$ and winning a match at Bengaluru is $\frac{6}{7}$; the probability of the team winning at least one match is____

Solu:

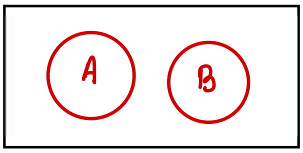
$P(K) = \frac{3}{5}$, $P(B) = \frac{6}{7}$, $P(K \cap B) = \frac{3}{5} \times \frac{6}{7} = \frac{18}{35}$

$P(K \cup B) = P(K) + P(B) - P(K \cap B)$

$= \frac{3}{5} + \frac{6}{7} - \frac{18}{35}$

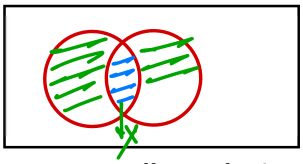
$= \frac{21+30-18}{35} = \frac{33}{35}$

Probability of happening of exactly One of A & B



Mutually exclusive

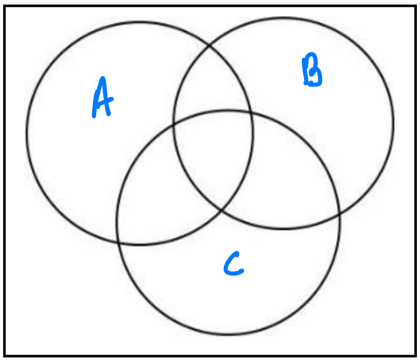
$P(A \cup B) = P(A) + P(B)$



Not Mutually exclusive

$P(A \cup B) = P(A) + P(B) - 2P(A \cap B)$

In Case of 3 Events



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Examples on Category 1 & 2 of GAT

Example: A dice is rolled. What is probability that a number 1 or 6 may appear on the face?

Solution:

1, 2, 3, 4, 5, 6

CDP — F/T

GAT

$P(1 \cup 6) = P(1) + P(6) - P(1 \cap 6)$

$= \frac{1}{6} + \frac{1}{6} - 0 = \frac{2}{6} = \frac{1}{3}$

(KDR)

Example: If probability of the horse A winning the race is $\frac{1}{5}$ and the probability of horse B winning the same race is $\frac{1}{6}$, what is the probability that one of the horses will win the race?

Solution:

$P(A) = 1/5$

$P(B) = 1/6$

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{1}{5} + \frac{1}{6} - 0$

$= \frac{6+5}{30} = \frac{11}{30}$ Ans.

Example: A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or a bolt?

Solution:

Rusted

50 Bolts → 25
150 Nuts → 75
200 | 100

C.O.P $\Rightarrow \frac{125}{200}$

GAT

$P(R \cup B) = P(R) + P(B) - P(R \cap B)$

$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{125}{200}$

Example: A bag contains 12 balls which are numbered from 1 to 12. If a ball is selected at random, what is the probability that the number of the ball will be a multiple of 5 or 6?

Solution: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

C.O.P $= F/T$

$\Rightarrow 4/12$

GAT

$P(5 \cup 6) = P(5) + P(6) - P(5 \cap 6)$

$= 2/12 + 2/12 - 0 = 4/12$

$(LOR)^2$

Example: Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are kings?

Solution: $26 + 2 = 28$

C.O.P

$P(A) = \frac{{}^{28}C_2}{{}^{52}C_2}$ WRONG

GAT

$P(R \cup K) = P(R) + P(K) - P(R \cap K)$

$= \frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{{}^{2}C_2}{{}^{52}C_2} = \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} = \frac{330}{1326}$

Example: Probability that Hameed passes in mathematics is $\frac{2}{3}$ and the probability that he passes in English is $\frac{4}{9}$. If the probability of passing both courses is $\frac{1}{4}$ what is the probability that Hameed will pass in at least one of these subjects?

Solution:

$P(M) = 2/3$

$P(E) = 4/9$

$P(M \cap E) = 1/4$

$P(M \cup E) = P(M) + P(E) - P(M \cap E)$

$= \frac{2}{3} + \frac{4}{9} - 1/4$

$= \frac{24 + 16 - 9}{36} = \frac{31}{36}$

Example: There are three persons A, B and having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

Solution:

$P(A) = 0.80$ $\quad P(A\cap C) = 0.48$

$P(B) = 0.60$ $\quad P(A\cap B\cap C) = 0.26$

$P(C) = 0.50$

$P(A\cap B) = 0.46$

$P(B\cap C) = 0.32$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$$= 0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26$$

$$\boxed{= 0.9}$$

Example The probability that a person visiting a dentist will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24. The probability that he will have his teeth cleaned or a cavity filled is 0.60. What is the probability that a person visiting a dentist will have his teeth cleaned and cavity filled?

Solution:

$P(TC) = 0.44$

$P(CV) = 0.24$

$P(TC \cup CV) = 0.60$

$P(TC \cap CV) = ?$

$$P(TC\cup CV) = P(TC) + P(CV) - P(TC\cap CV)$$

$$0.60 = 0.44 + 0.24 - P(x)$$

$$\boxed{P(x) = 0.08}$$ Ans.

3 Where Question Demands **At least One Event** to Occur for ***INDEPENDENT EVENTS***

No worry as they are asking At Least One Event So We Will apply **GAT** → Problem is We Don't Know $P(A\cap B)$ → We Can Solve Only After Understanding **GMT**

After Understand

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cap B) \to P(A).P(B)$

If events are Independent.

Shortcut

$1 - \left[\text{Probability that I don't want}\right]$

Examples on Category 3 of GAT

MTP-Nov-22

Example Ram is known to hit a target in 2 out of 3 shots whereas Shyam is known to hit the same target in 5 out of 11 shots. What is the probability that the target would be hit if they both try?

(a) $\frac{9}{11}$ (b) $\frac{6}{11}$ (c) $\frac{10}{33}$ (d) $\frac{3}{11}$

Solution:

MTP-Jun-22

Example A husband and a wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 3/5 and that of wife's selection is 1/5. Then the probability that only one of them is selected is:

(a) 16/25 (b) 17/25 (c) 14/25 (d) none of these

Solution:

Example: A problem in probability was given to three CA students A, B and C whose chances of solving it are 1/3, 1/5 and 1/2 respectively. What is the probability that the problem would be solved?

Solution:

Shortcut

General Multiplication Theorem (GMT)

When we want Both the events to Occur Simultaneously. [∩]

AND

Independent Event

When happening of one event **does not affect** the happening or non-happening of another event, then it is known as Independent Event

With Replacement

When the Happening of ONE EVENT ***Does not*** *affect the* Sample Space *for another event then events are known as* ***Independent Events.***

$$P(A \cap B) = P(A) \times P(B)$$

Dependent Event

When happening of one event **affect** the happening or non-happening of another event, then it is known as Independent Event

When the Happening of ONE Event Changes *the* Sample Space. *for another event then events are known as* ***Dependent Events.***

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$$

Let's Understand with Examples Below

S.no	Situation	Type of Event
1.	Arun & Tarun appear for an interview for two vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5	Independent
2.	The probability of a cricket team winning match at Kanpur is 2/5 and winning match at Delhi is 1/7 what is the Probability of winning both the match?	Independent
3.	If a speaks 75% of truth and B speaks 80% of truth.	Independent
4.	Probability that Hameed passes in mathematics is $\frac{2}{3}$ and the probability that he passes in English is $\frac{4}{9}$.	Independent
5.	Two cards are drawn from a pack of 52 cards One by One without replacement. What is the probability that either both are King?	Dependent
6.	Two cards are drawn from a pack of 52 cards One by One with replacement. What is the probability that either both are King?	Independent
7.	The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60.	Independent.
8.	A bag contains 5 white, 7 red and 8 black balls. Four balls are drawn one by one with replacement	Independent.
9.	A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement.	Dependent.

Examples on GMT

Example A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?

10W
15B

$$P(W \cap B) = P(W) \times P\left(\frac{B}{W}\right) = \frac{10}{25} \times \frac{15}{24} = \frac{150}{600}$$

Example A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement, find the probability of getting all white balls.

5W
7R
8B

$$P(W_1 \cap W_2 \cap W_3 \cap W_4) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} \times \frac{2}{17} = \frac{120}{116280}$$

Example Find the probability of drawing a diamond card in each of the two consecutive draws from a well shuffled pack of cards, if the card drawn is not replaced after the first draw.

$$P(D_1 \cap D_2) = P(D_1) \times P\left(\frac{D_2}{D_1}\right) = \frac{13}{52} \times \frac{12}{51} = \frac{156}{2652}$$

Example A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made
(i) With replacement
(ii) without replacement.
The probability that the first draw would produce white balls and the second draw would produce black balls are respectively

5W
7B

With Replacement

$$P(W \cap B) = P(W) \times P(B)$$

$$= \frac{^5C_3}{^{12}C_3} \times \frac{^7C_3}{^{12}C_3}$$

$$= \frac{\cancel{10}}{\cancel{220}_{44}} \times \frac{\cancel{35}^{7}}{\cancel{220}} = \frac{7}{968}$$

Without Replacement

$$P(W \cap B) = P(W) \times P\left(\frac{B}{W}\right)$$

$$= \frac{^5C_3}{^{12}C_3} \times \frac{^7C_3}{^9C_3}$$

$$= \frac{\cancel{10}}{\cancel{220}} \times \frac{\cancel{35}^{5}}{\cancel{84}_{12}} = \frac{5}{264}$$

Conclusion from above Question

Order of the Draw is **Mentioned** in the Question	Order of the Draw is **Not Mentioned** in the Question
	*Solve **Multiple Case** that Satisfy the **Demands** of the Questions* — And Add them to get Total Prob.

Example: A bag contains 5 blue and 4 red balls. If two balls are drawn random one by one without replacement, find the probability of getting first ball as blue and second ball as red?

Sol:

$\boxed{\begin{matrix} 5B \\ 4R \end{matrix}}$

Example: A bag contains 5 blue and 4 red balls. If two balls are drawn random one by one without replacement, find the probability of getting one ball is blue and one ball is red?

Sol:

$\boxed{\begin{matrix} 5B \\ 4R \end{matrix}}$

Total Probability + GMT

Example: From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and are then multiplied. The probability that the product of the chosen numbers will be positive number is

(a) $\frac{409}{1001}$ (b) $\frac{70}{1001}$ (c) $\frac{505}{1001}$ (d) $\frac{420}{1001}$

$6 \to +ve$, $8 \to -ve$

$++++$ $\quad ----$ $\quad ++--$

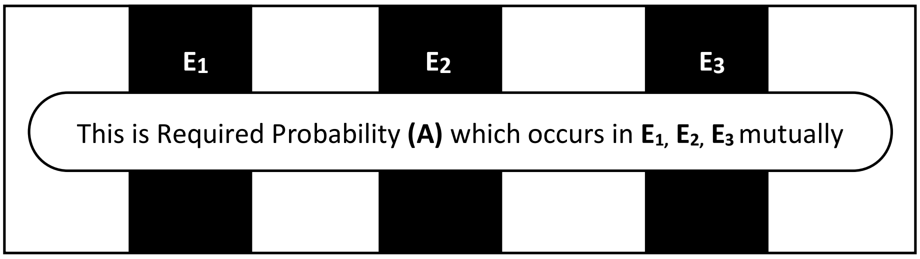
$$\frac{{}^6C_4}{{}^{14}C_4} + \frac{{}^8C_4}{{}^{14}C_4} + \frac{{}^6C_2 \times {}^8C_2}{{}^{14}C_4} \Rightarrow \frac{15}{1001} + \frac{70}{1001} + \frac{420}{1001}$$

$$= \frac{505}{1001}$$

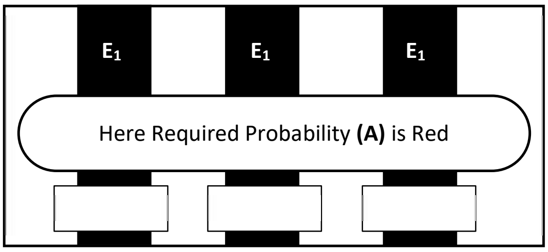
Total Probability

If $E_1, E_2, E_3, \ldots$ are mutually exclusive and exhaustive events and **A is any event which occurs with E_1 E_1, E_2, E_3 …. Then**

$$P(A) = P(E_1)\times P(A/E_1) + P(E_2)\times P(A/E_2) + P(E_3)\times P(A/E_3)$$



Example: A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. from the selected bag, one ball is drawn. Find the probability that the ball drawn is red.



$\boxed{4R \; 3B}$ $\boxed{2R \; 4B}$

$P(B_1 \cap R) + P(B_2 \cap R)$

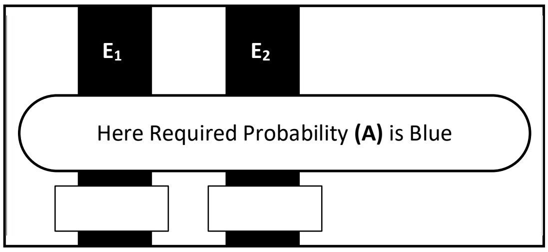
$P(B_1) \times P(R) + P(B_2) \times P(R)$

$\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{2}{6}$

$4/14 + 2/12 = \frac{48+28}{168} = \frac{76}{168}$

$(KDR)^2$

Example: A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from first bag without noticing its color is put in second bag. A ball is then drawn from the second bag. Find the probability that the ball drawn is blue



$\boxed{6R \; 5B}$ $\boxed{5R \; 8B}$

$E_1 + E_2$

Conditional Probability

Conditional probability refers to the chances of occurs of **One Event** given **that another event has also occurred.**

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Important Note:

Example: In a class 40% students read Mathematics, 25 % Biology and 15 % both Mathematics and Biology. One student is select at random. The probability that he reads Mathematics if it is known that he reads Biology is

Solution:

$P(M) = 0.40$

$P(B) = 0.25$

$P(M \cap B) = 0.15$

$$P\left(\frac{M}{B}\right) = \frac{P(M \cap B)}{P(B)} = \frac{0.15}{0.25}$$

$$\boxed{= 0.6 \text{ Ans.}}$$

Example: The Probability of a student passing in science is $^4/_5$ and the probability of the student passing in both science & maths is $^1/_2$. What is the probability of that student passing in maths knowing that he passed in science?

Solution:

$P(S) = 4/5$

$P(S \cap M) = 1/2$

$$P\left(\frac{M}{S}\right) = \frac{P(M \cap S)}{P(S)} = \frac{1/2}{4/5}$$

$$= \frac{1}{2} \times \frac{5}{4} = \frac{5}{8} \text{ Ans.}$$

Expected Value

Expected Value [E(x)] or Mathematical Expectation of a random variable is the sum of products of the different values taken by the random variable and the corresponding probabilities.

$E(x) = \sum P_i x_i$

$E(x^2) = \sum P_i x_i^2$

Variance

Variance $(\sigma^2) = E(x-\mu)^2$

*** $\boxed{V(x) = E(x^2) - [E(x)]^2}$

Important Highlights

1. Expectation of constant K = K
2. E[x +y] = E[x] + E[y]
3. E[kx] = kE[x]
4. E[x.y] = E[x]. E[y]

Example: If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is

x :-	0	1	2
$P(x)$:-	0.30	0.50	0.20

$E(x) = \sum P_i x_i$

$\sum P_i x_i$:- $0 + 0.50 + 0.40 = 0.90$

Example: A packet of 10 electronic components is known to include 3 defectives. If 4 components are selected from the packet at random, what is the expected value of the number of defective?

$^{10}C_4 = 210$

$10 \begin{cases} 3D \\ 7G \end{cases}$

x :-	1	2	3
$P(x)$:-	$\frac{^3C_1 \times ^7C_3}{^{10}C_4} = 0.5$	$\frac{^3C_2 \times ^7C_2}{^{10}C_4} = 0.3$	$\frac{^3C_3 \times ^7C_1}{^{10}C_4} = 0.0\overline{3}333$
$P_i x_i$	$= 0.5 +$	$0.6 +$	0.0999

$\sum P_i x_i = 1.199 \simeq 1.2$

Example: If two random variables x and y are related as y = -3x + 4 and standard deviation of x is 2, then the standard deviation of y is

$y = -3x + 4$

$\sigma_x = 2$

$\sigma_y = \_\_\_$

$\sigma_y = |b| . \sigma_x$

$= 3 \times 2$

$\boxed{\sigma_y = 6}$

Example The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statements

$TS = 220$

	A	B	C
x :-	60	70	90
$P(x)$:-	0.80	0.70	0.90

$E(x) = \sum P_i x_i = 48 + 49 + 81 = 178$

$\boxed{E(x) = 178}$

Example The probability distribution of a random variable x is given below:

x:	1	2	4	5	6
p:	0.15	0.25	0.20	0.30	0.10

What is the standard deviation of x?

$E(x) = \sum P_i x_i \Rightarrow 3.55$

$E(x^2) = \sum P_i x_i^2 \Rightarrow 15.45$

$V(x) = E(x^2) - [E(x)]^2$

$= 15.45 - (3.55)^2$

$V(x) = 2.8475 \rightarrow \sigma^2$

$\boxed{\sigma = 1.68}$ Ans.

Other Important Concepts

Concept of 3 Dice

The Question Based on 3 Dices are Generally Very Lengthy Question because Sample Space of 3 Dices is Vey Huge **(6^3=216)** which is not possible to draw in exam. But ICAI have asked Question Based on 3 Dices Many Time () So, Lets Discuss this Variety with Tricks

$$P(\text{Sum}) = \frac{\text{Favorable Outcome}}{216}$$

Trick to Favourable no Outcomes

Sum	Cases	Sum
10	27	11
9	25	12
8	21	13
7	15	14
6	10	15
5	6	16
4	3	17
3	1	18

Example: 3 dices is rolled what is the probability of Sum 6?

$P(R) = \frac{F}{T} = \frac{10}{216}$ Ans.

Example: 3 dices is rolled what is the probability of Sum 13?

$P(R) = \frac{F}{T} = \frac{21}{216}$ Ans.

(KDR)²

Nov-2020

Example: When 3 dice are rolled simultaneously the probability of a number on the third die is greater than the sum of the numbers on two dice.

(a) 12/216 **(b)** 36/216 **(c)** 48/216 **(d)** 20/216

_ _ 3   Sum = 2   Fav:- 1

_ _ 4   Sum = 2, 3   1 + 2

_ _ 5   Sum = 2, 3, 4   1 + 2 + 3

_ _ 6   Sum = 2, 3, 4, 5.   1 + 2 + 3 + 4 =

$\frac{\text{Fav}}{\text{Total}}$

$\frac{20}{216}$

Jan-2021

Example: Three identical and balanced dice are rolled. The probability that the same number will appear on each of them is.

(a) $\frac{1}{6}$ **(b)** $\frac{1}{18}$ **(c)** $\frac{1}{36}$ **(d)** $\frac{1}{24}$

(1,1,1) (2,2,2), (3,3,3) (4,4,4) (5,5,5), (6,6,6)

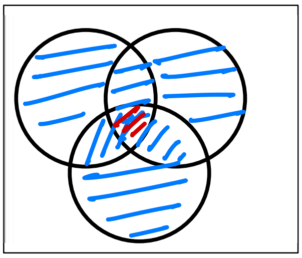
$= \frac{6}{216} = 1/36$ Ans.

Let's Discuss Most Important Question Student's Usually Get Confused

Question: There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?

1

What is the probability that at **least one** will survive another five years?



$P(A) = 0.7$
$P(B) = 0.4$
$P(C) = 0.2$

$1 - [\text{All will die}]$

$1 - [\bar{A} \cap \bar{B} \cap \bar{C}]$

$1 - [0.3 \times 0.6 \times 0.8]$

$\downarrow$

$\boxed{0.856}$

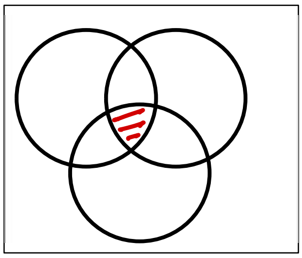
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= 0.7 + 0.4 + 0.2 - 0.28 - 0.08 - 0.14 + 0.056$

$\boxed{= 0.856}$

2

What is the probability that **all three** will survive another five years?



$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

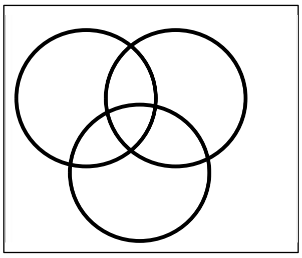
$= 0.7 \times 0.4 \times 0.2$

$\boxed{= 0.056}$

3

What is the probability that **at least two** of them would survive another five years?

How we Think!



How we Should Think!